Fundamentals of Aerodynamics

Sixth Edition

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present and past engineering literature written in the English engineering system, literature that will be used well into the future. The modern engineering student must be bilingual in these units, and must feel comfortable with both systems. For this reason, although many of the worked examples and end-of-the-chapter problems in this book are in the SI units, some are in the English engineering system of units. You are encouraged to join this bilingual spirit and to work to make yourself comfortable in both systems.

1.5 AERODYNAMIC FORCES AND MOMENTS

At first glance, the generation of the aerodynamic force on a giant Boeing 747 may seem complex, especially in light of the complicated three-dimensional flow field over the wings, fuselage, engine nacelles, tail, etc. Similarly, the aerodynamic resistance on an automobile traveling at 55 mi/h on the highway involves a complex interaction of the body, the air, and the ground. However, in these and all other cases, the aerodynamic forces and moments on the body are due to only two basic sources:

1. *Pressure distribution* over the body surface
2. *Shear stress distribution* over the body surface

No matter how complex the body shape may be, the aerodynamic forces and moments on the body are due entirely to the above two basic sources. The *only* mechanisms nature has for communicating a force to a body moving through a fluid are pressure and shear stress distributions on the body surface. Both pressure p and shear stress τ have dimensions of force per unit area (pounds per square foot or newtons per square meter). As sketched in Figure 1.15, p acts *normal* to the surface, and τ acts *tangential* to the surface. Shear stress is due to the "tugging action" on the surface, which is caused by friction between the body and the air (and is studied in great detail in Chapters 15 to 20).

The net effect of the p and τ distributions integrated over the complete body surface is a resultant aerodynamic force R and moment M on the body, as sketched in Figure 1.16. In turn, the resultant R can be split into components, two sets of

There is no explanation of this transformation from p and τ to R.

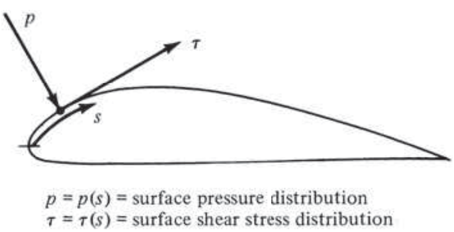

Figure 1.15 Illustration of pressure and shear stress on an aerodynamic surface.

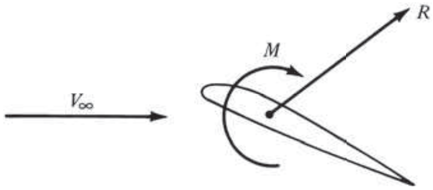

Figure 1.16 Resultant aerodynamic force and moment on the body.

There is no explanation of how the resultant aerodynamic force, R, is oriented in this direction.

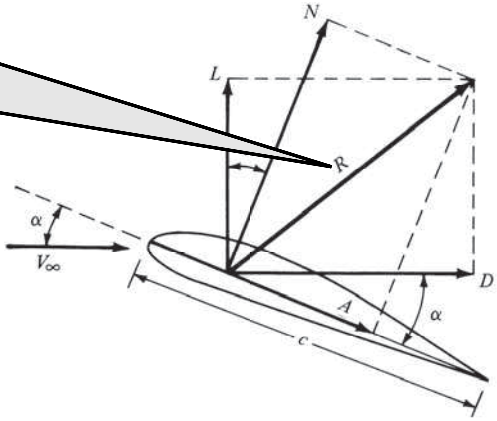

Figure 1.17 Resultant aerodynamic force and the components into which it splits.

which are shown in Figure 1.17. In Figure 1.17, V_∞ is the *relative wind,* defined as the flow velocity far ahead of the body. The flow far away from the body is called the *freestream,* and hence V_∞ is also called the freestream velocity. In Figure 1.17, by definition,

$$L \equiv \text{lift} \equiv \text{component of } R \text{ perpendicular to } V_\infty$$

$$D \equiv \text{drag} \equiv \text{component of } R \text{ parallel to } V_\infty$$

The chord c is the linear distance from the leading edge to the trailing edge of the body. Sometimes, R is split into components perpendicular and parallel to the chord, as also shown in Figure 1.17. By definition,

$$N \equiv \text{normal force} \equiv \text{component of } R \text{ perpendicular to } c$$

$$A \equiv \text{axial force} \equiv \text{component of } R \text{ parallel to } c$$

The angle of attack α is defined as the angle between c and V_∞. Hence, α is also the angle between L and N and between D and A. The geometrical relation between these two sets of components is, from Figure 1.17,

$$L = N\cos\alpha - A\sin\alpha \tag{1.1}$$

$$D = N\sin\alpha + A\cos\alpha \tag{1.2}$$

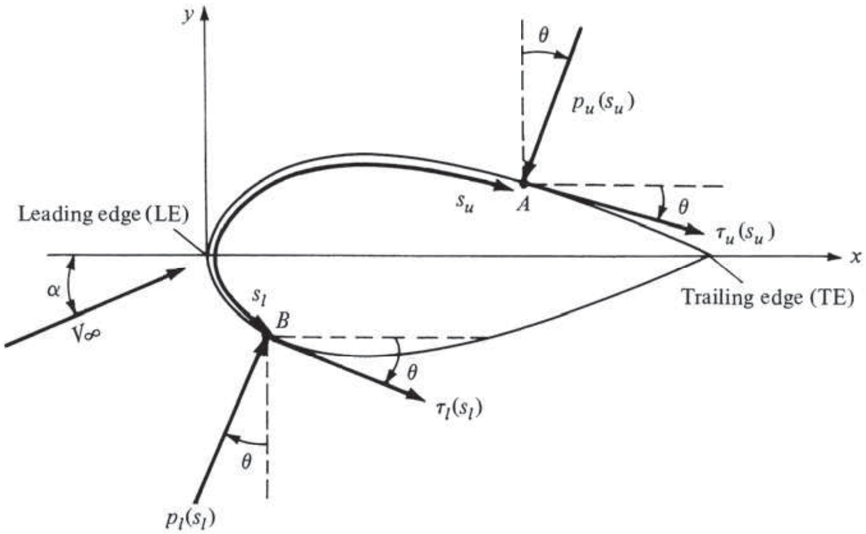

Figure 1.18 Nomenclature for the integration of pressure and shear stress distributions over a two-dimensional body surface.

Let us examine in more detail the integration of the pressure and shear stress distributions to obtain the aerodynamic forces and moments. Consider the two-dimensional body sketched in Figure 1.18. The chord line is drawn horizontally, and hence the relative wind is inclined relative to the horizontal by the angle of attack α. An xy coordinate system is oriented parallel and perpendicular, respectively, to the chord. The distance from the leading edge measured along the body surface to an arbitrary point A on the upper surface is s_u; similarly, the distance to an arbitrary point B on the lower surface is s_l. The pressure and shear stress on the upper surface are denoted by p_u and τ_u, both p_u and τ_u are functions of s_u. Similarly, p_l and τ_l are the corresponding quantities on the lower surface and are functions of s_l. At a given point, the pressure is normal to the surface and is oriented at an angle θ relative to the perpendicular; shear stress is tangential to the surface and is oriented at the same angle θ relative to the horizontal. In Figure 1.18, the sign convention for θ is positive when measured *clockwise* from the vertical line to the direction of p and from the horizontal line to the direction of τ. In Figure 1.18, all thetas are shown in their positive direction. Now consider the two-dimensional shape in Figure 1.18 as a cross section of an infinitely long cylinder of uniform section. A unit span of such a cylinder is shown in Figure 1.19. Consider an elemental surface area dS of this cylinder, where $dS = (ds)(1)$ as shown by the shaded area in Figure 1.19. We are interested in the contribution to the total normal force N' and the total axial force A' due to the pressure and shear stress on the elemental area dS. The primes on N' and A' denote force per unit span. Examining both Figures 1.18 and 1.19, we see that the elemental normal and axial forces acting on the elemental surface dS on the

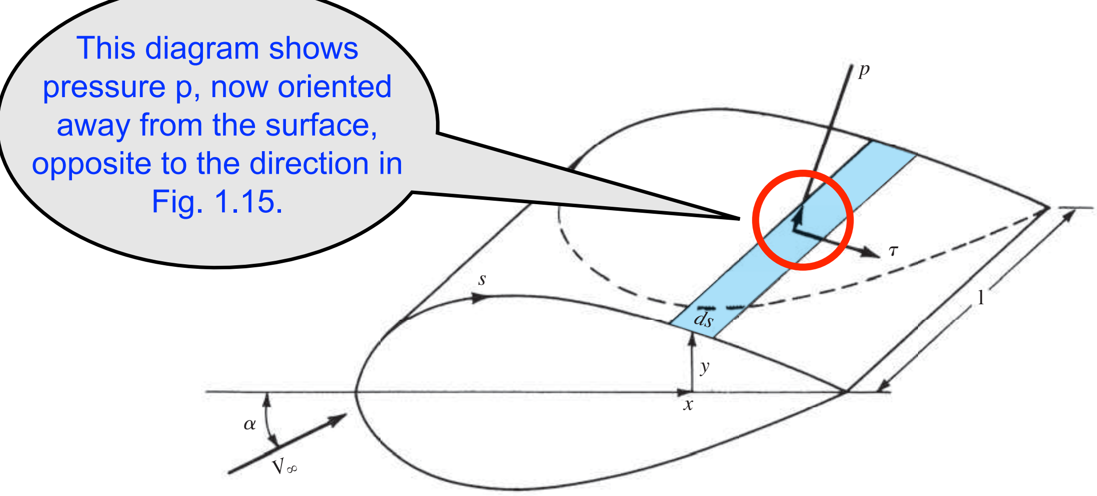

Figure 1.19 Aerodynamic force on an element of the body surface.

upper body surface are

$$dN'_u = -p_u ds_u \cos\theta - \tau_u ds_u \sin\theta \tag{1.3}$$

$$dA'_u = -p_u ds_u \sin\theta + \tau_u ds_u \cos\theta \tag{1.4}$$

On the *lower* body surface, we have

$$dN'_l = p_l ds_l \cos\theta - \tau_l ds_l \sin\theta \tag{1.5}$$

$$dA'_l = p_l ds_l \sin\theta + \tau_l ds_l \cos\theta \tag{1.6}$$

In Equations (1.3) to (1.6), the positive directions of N' and A' are those shown in Figure 1.17. In these equations, the positive clockwise convention for θ must be followed. For example, consider again Figure 1.18. Near the leading edge of the body, where the slope of the upper body surface is positive, τ is inclined upward, and hence it gives a positive contribution to N'. For an upward inclined τ, θ would be counterclockwise, hence negative. Therefore, in Equation (1.3), $\sin\theta$ would be negative, making the shear stress term (the last term) a positive value, as it should be in this instance. Hence, Equations (1.3) to (1.6) hold in general (for both the forward and rearward portions of the body) as long as the above sign convention for θ is consistently applied.

The total normal and axial forces *per unit span* are obtained by integrating Equations (1.3) to (1.6) from the leading edge (LE) to the trailing edge (TE):

$$N' = -\int_{\mathrm{LE}}^{\mathrm{TE}} (p_u \cos\theta + \tau_u \sin\theta)\, ds_u + \int_{\mathrm{LE}}^{\mathrm{TE}} (p_l \cos\theta - \tau_l \sin\theta)\, ds_l \tag{1.7}$$

$$A' = \int_{\mathrm{LE}}^{\mathrm{TE}} (-p_u \sin\theta + \tau_u \cos\theta)\, ds_u + \int_{\mathrm{LE}}^{\mathrm{TE}} (p_l \sin\theta + \tau_l \cos\theta)\, ds_l \tag{1.8}$$

Figure 1.20 Sign convention for aerodynamic moments.

In turn, the total lift and drag per unit span can be obtained by inserting Equations (1.7) and (1.8) into (1.1) and (1.2); note that Equations (1.1) and (1.2) hold for forces on an arbitrarily shaped body (unprimed) and for the forces per unit span (primed).

The aerodynamic moment exerted on the body depends on the point about which moments are taken. Consider moments taken about the leading edge. By convention, moments that tend to increase α (pitch up) are positive, and moments that tend to decrease α (pitch down) are negative. This convention is illustrated in Figure 1.20. Returning again to Figures 1.18 and 1.19, the moment per unit span about the leading edge due to p and τ on the elemental area dS on the upper surface is

$$dM'_u = (p_u \cos\theta + \tau_u \sin\theta)x\, ds_u + (-p_u \sin\theta + \tau_u \cos\theta)y\, ds_u \tag{1.9}$$

On the bottom surface,

$$dM'_l = (-p_l \cos\theta + \tau_l \sin\theta)x\, ds_l + (p_l \sin\theta + \tau_l \cos\theta)y\, ds_l \tag{1.10}$$

In Equations (1.9) and (1.10), note that the same sign convention for θ applies as before and that y is a positive number above the chord and a negative number below the chord. Integrating Equations (1.9) and (1.10) from the leading to the trailing edges, we obtain for the moment about the leading edge per unit span

$$\begin{aligned} M'_{\mathrm{LE}} = & \int_{\mathrm{LE}}^{\mathrm{TE}} [(p_u \cos\theta + \tau_u \sin\theta)x - (p_u \sin\theta - \tau_u \cos\theta)y]\, ds_u \\ & + \int_{\mathrm{LE}}^{\mathrm{TE}} [(-p_l \cos\theta + \tau_l \sin\theta)x + (p_l \sin\theta + \tau_l \cos\theta)y]\, ds_l \end{aligned} \tag{1.11}$$

In Equations (1.7), (1.8), and (1.11), θ, x, and y are known functions of s for a given body shape. Hence, if p_u, p_l, τ_u, and τ_l are known as functions of s (from theory or experiment), the integrals in these equations can be evaluated. Clearly, Equations (1.7), (1.8), and (1.11) demonstrate the principle stated earlier, namely, *the sources of the aerodynamic lift, drag, and moments on a body are the pressure and shear stress distributions integrated over the body.* A major goal of theoretical aerodynamics is to calculate $p(s)$ and $\tau(s)$ for a given body shape and freestream conditions, thus yielding the aerodynamic forces and moments via Equations (1.7), (1.8), and (1.11).

As our discussions of aerodynamics progress, it will become clear that there are quantities of an even more fundamental nature than the aerodynamic forces and moments themselves. These are *dimensionless force and moment coefficients,* defined as follows. Let ρ_∞ and V_∞ be the density and velocity, respectively, in

Skip 261 pages (of complex explanations) to Part 2, page 284,

The source of the aerodynamic force is explained in Sect 1.5!!!!

obtain the lift—is the essence of the *circulation theory of lift* in aerodynamics. Its development at the turn of the twentieth century created a breakthrough in aerodynamics. However, let us keep things in perspective. The circulation theory of lift is an *alternative* way of thinking about the generation of lift on an aerodynamic body. Keep in mind that the true physical sources of aerodynamic force on a body are the pressure and shear stress distributions exerted on the surface of the body, as explained in Section 1.5. The Kutta-Joukowski theorem is simply an alternative way of expressing the *consequences* of the surface pressure distribution; it is a mathematical expression that is consistent with the special tools we have developed for the analysis of inviscid, incompressible flow. Indeed, recall that Equation (3.140) was derived in Section 3.15 by integrating the pressure distribution over the surface. Therefore, it is not quite proper to say that circulation "causes" lift. Rather, lift is "caused" by the net imbalance of the surface pressure distribution, and circulation is simply a defined quantity determined from the same pressures. The relation between the surface pressure distribution (which produces lift L') and circulation is given by Equation (3.140). However, in the theory of incompressible, potential flow, it is generally much easier to determine the circulation around the body rather than calculate the detailed surface pressure distribution. Therein lies the power of the circulation theory of lift.

Consequently, the theoretical analysis of lift on two-dimensional bodies in incompressible, inviscid flow focuses on the calculation of the circulation about the body. Once Γ is obtained, then the lift per unit span follows directly from the Kutta-Joukowski theorem. As a result, in subsequent sections we constantly address the question: How can we calculate the circulation for a given body in a given incompressible, inviscid flow?

3.17 NONLIFTING FLOWS OVER ARBITRARY BODIES: THE NUMERICAL SOURCE PANEL METHOD

In this section, we return to the consideration of nonlifting flows. Recall that we have already dealt with the nonlifting flows over a semi-infinite body and a Rankine oval and both the nonlifting and the lifting flows over a circular cylinder. For those cases, we added our elementary flows in certain ways and discovered that the dividing streamlines turned out to fit the shapes of such special bodies. However, this indirect method of starting with a given combination of elementary flows and seeing what body shape comes out of it can hardly be used in a practical sense for bodies of arbitrary shape. For example, consider the airfoil in Figure 3.37. Do we know in advance the correct combination of elementary flows to synthesize the flow over this specified body? The answer is no. Rather, what we want is a direct method; that is, let us *specify* the shape of an arbitrary body and *solve* for the distribution of singularities which, in combination with a uniform stream, produce the flow over the given body. The purpose of this section is to present such a direct method, limited for the present to nonlifting flows. We consider a numerical method appropriate for solution on a high-speed digital computer. The technique